Elasticity in the physics of sports

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July 16, 2012

Abstract

Elasticity plays a role in many instances of sports. The aim of this introductory paper is to sketch a general viewpoint that would help building links between the different contributions to the conference involving elasticity.

1 Introduction

The starting point of this paper considers the management of energy during walking. The mechanism of the leg-walking makes use of the gravity to reduce energy costs: we need a way to store energy to compensate for the fact that the kinetic energy is not constant during the periodic motion. From walking we move to running and we will see how inner elasticity of the body will play the role that gravity plays for walking. Having shown this way the presence and importance of elasticity in the body, I will discuss athletic disciplines with strong shocks: high jump, long jump and pole vault for instance. We will see how energy can be transformed into performance by using inner elasticity. Pole vault allows the use of an elastic device outside the body to compensate for limitations of inner elasticity. In the last section, I lump other elastic phenomena in sports of interest to the discussion.

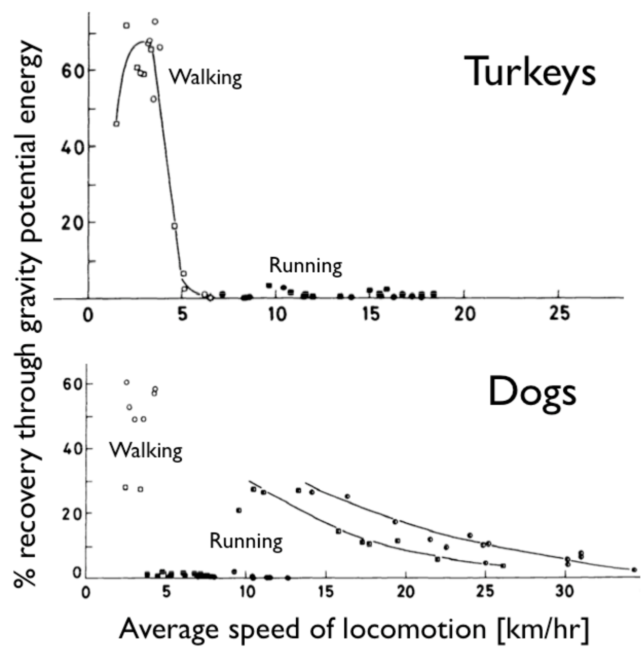

Figure 1: Recovery of mechanical energy in walking and running as a function of speed for dog and turkey. "Recovery" indicates the extent of mechanical energy reutilization through the shift between potential and kinetic energy as in a pendulum: elastic energy is not taken into account here. This recovery is maximal in walking, intermediate in gallop and minimal in running. Figure from [1].

2 Moving at constant energy

To walk forward we need to set our limbs in motion—change the shape of our body—and it would be useful if just like a wheel, we could do this at constant kinetic energy. Unfortunately, we need to accellerate and decelerate each of our legs and arms: kinetic energy is not constant during each period of our walk. Without the help of a specific mechanism, we would have to produce each time this kinetic energy from our chemical energy stores, and then degrade it to heat to slow down the limb. Fortunately, the motion of our legs is made such that the kinetic energy is transfered to potential energy by raising the center of mass of the leg. This way, we can view the leg as analogous to a pendulum. The attraction force from the planet on which we walk is thus a key element in saving energy, see for

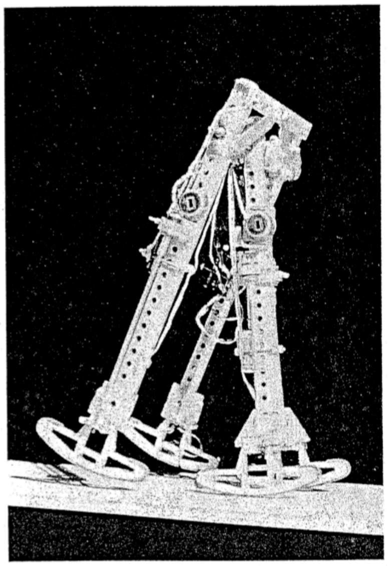

Figure 2: Biped used for experiment on two-dimensional gravity-powered walking. Figure from [3]. These studies are ideal illustrations of motion at almost constant energy. We make use of gravity potential energy and elastic potential energy in our bodies to approach this ideal of a motion at constant energy.

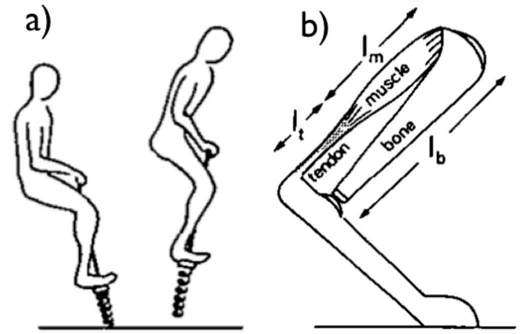

Figure 3: Understanding the role played by the elasticity of muscle/tendons by taking the example of the bounce of the kangaroo. a) A child jumping on a pogo stick. b) The equivalent to the spring of the pogo stick inside the leg of a kangaroo. Figure adapted from [4].

instance [2]. Figure 1 shows experimental data of recovery of energy for dogs and turkey, two animals with apparently different walking strategies: one biped bird and a quadruped mammal. We see in this figure as well that the pendulum mechanism of energy recovery in not active in running, we will see in later sections that elasticity will play the role of a second store of energy.

Interesting illustrations of this principle of energy re-utilization through a second store for energy can be found in the studies of passive walking: a legged device is built to move forward by stepping with no external energy except that which can be extracted from a gently sloping floor. The role of the slope is to compensate for the small amount of energy dissipation through friction. Here, the periodic motion of walking must really make use of exchange of kinetic and potential energy, see for a review [3]. Figure 2 shows a picture of one such device walking down an incline. Here, there is no muscular action, such that the structure: oscillation period, rocking and stepping phases must be thought and designed such that kinetic energy of motion be exactly restored using gravity forces and also possibly elastic springs.

3 Storing energy in elastic strain

When running, this mechanism of using gravity potential energy is not as useful, considering the higher frequencies involved. The motion of the limbs is fast and we need to use an other type of potential energy. This is where elasticity comes into play. As a first example, we can think of the hoping kangaroo: it moves forward just like a bouncing ball: the legs act like springs. Where are the springs? It is the muscles and tendons which are the essential elastic elements which can store kinetic energy into elastic strain of elongation and restore it back when bouncing. Figure 3 illustrates this analogy. For a running athlete, the process is very similar: limbs are accelerated and decelerated with respect to the center of mass of the body in forward translation. The legs: knees and ankles are the structural elements made such as to be able to use the elongation of muscles and tendons to bounce. The two basic articles on that are [1, 4].

When looking at specific sport applications, we can see how this mechanism is declined, depending on frequencies and magnitude of the forces involved. The contribution of G. Laffaye to the conference, "Human body as a spring-mass system", [5] discusses the particular stiffness of the leg needed for running very fast on a short distance. When sprinting, the duration of the step is extremely short—about 0.1 second—so the leg must act as an extremely stiff spring in order to have the phase of extension and contraction at the right time. A less trained sprinter with softer muscle/tendon system will need a longer time with the foot on the ground, otherwise he will not be able to use this elastic spring mechanism. The recoil of the leg spring must happen in harmony with the timing of the motion.

The general view on sports is that performance require skill, but also a large power, which is the object of intensive training. By discussing the role of elasticity, we see emerging the notion that not only the *active properties* of the muscles are important: voluntary contraction, but as well the *passive properties* of the muscles/tendons. The stiffness of muscles and tendons can be increased using *plyometric* exercise. These are a type of exercises designed to produce fast and powerful movements: it does not only play the role of increasing the power in the muscles, but also of stiffening them: it modifies their passive dynamics. This way, training must as well serve to tune the stiffness of muscles/tendons to the masses in motion and times scales of the required moves. If the passive properties of the muscles are not fit, the muscular effort is useless: most of the power produced is lost. The body is a dynamic system: muscular actions are controls that need to account for the external elements like trajectory and speed, but this control action must be fit as well to internal dynamics. To give an order of magnitude of this softness/stiffness, studies on sprinting show that the passive stiffness of novices are typically half of that of expert sprinters, see [5].

In these two first sections, we have discussed how we can use potential energy to move more like a wheel: move at constant energy. It is interesting as well to note that there are also mechanisms by which we can mimic the gears of a car in order to tune the frequencies and forces that our muscles can produce to the desired speed. This was discussed in the contribution of A.E. Minetti, [6]. Walking and running make use of two different strategies for moving at constant energy, both involving to different extent gravity and elasticity. These two strategies can be thought as two gears to transform our chemical energy into motion. Four legged animals possess even more of these gears, for instance for horses: walk, trot and gallop.

4 Shocks and elasticity

Running is a most straightforward way for athletes to accumulate energy which they can then use for the different purposes of performance in their specific discipline. Let us consider for instance high jump. We have seen that in walking, we transfer periodically kinetic energy into height by raising the center of mass of the legs and restore it to kinetic energy. For jump disciplines we store kinetic energy into forward motion of the center of mass, and then try to transform as much as possible of this energy into height to clear a crossbar as high as possible in a final move. For this, the athlete need to redirect his velocity vector upward during his last step. This shock must be smoothed using the elastic properties of his body.

For sprint, we saw that the foot should rest on the floor as little as possible to run as fast as possible. For high jump, there is no intrinsic limitation for the time of this last contact. Just like a mass and spring system, this contact will be long with a force of low intensity if the spring is soft, but will be quick with a large force if the spring is stiff. It is interesting to see that both strategies can be found at the highest level of competition. In his contribution to the conference "High jump", G. Laffaye [7] shows the case of Donald Thomas, tall and soft with an Achilles tendon much longer than average. He uses very much the soft spring limit, with a large angular motion of the ankle. On the other hand, Stephan Holm, shorter than average high jumpers (1m80) trains specifically his stiffness, and shows almost no deformation during impulse.

The ultimate aim of using the passive elastic dynamics of the body here like for all disciplines with strong shocks is to avoid inelastic stretching of the muscles. This is analogous to walking down the stairs: the leg works against the weight while moving along the force of the weight. In this circumstance, the work exerted by the body is negative: energy is absorbed and degraded into heat. Thus, the athlete much train and strive to use has much of his passive elasticity during the shock. Reference [4] discusses the elasticity in the legs of running animals, and mentions that probably elasticity should be put into action in less obvious parts of the body, like for instance the back. The greyhound and the horse are good examples of using the back as an elastic device. We might think that the stiff strategy of Stephan Holm for high jump does not locate all of the elastic response in the Achilles tendon, but sets into action all of the body, striving to use the passive elastic behavior of the spine instead of active muscular work of the legs to oppose the deformations from the impulsion shock.

We may as well take the chance here to mention the contribution of A. Eddi, "High jump and pole vault: a classical tunneling ?" [8]: jumping high is not only about raising the center of gravity. Through deformation of the body, it is possible to clear a bar above the center of gravity, in a way which can be thought as analogous to the tunneling effect by which the wavelike behavior of elementary particles helps moving faster than light.

The aim of the long jumper is essentially different from the high jumper. The high jumper strives to redirect upward his velocity vector acquired through running. But he cannot run too fast because a shock too strong leads to inelastic dissipation and possibly injuries. The long jumper on the other hand makes direct use of his horizontal velocity. He should redirect to the vertical just that amount of his running energy which leads to the best parabola. For this, he uses the structure of his leg as a rigid pivot. The shock of this last impulse leads to a strong dissipation. The athlete faces a dilemma: a high take-off angle has the cost of a strong shock but helps making use for a longer time of the forward velocity during a longer parabola. The optimal angle is observed to be about 20 degrees, instead of the 45 degrees angle which leads to the longest ballistic flight [9].

For pole vault, there is an additional element in the discussion. The aim of the pole vaulter is essentially the same as the high jumper, except for the fact that he is allowed to use an elastic device outside of his body: the pole. At early times of the discipline, the pole was made of wood and very

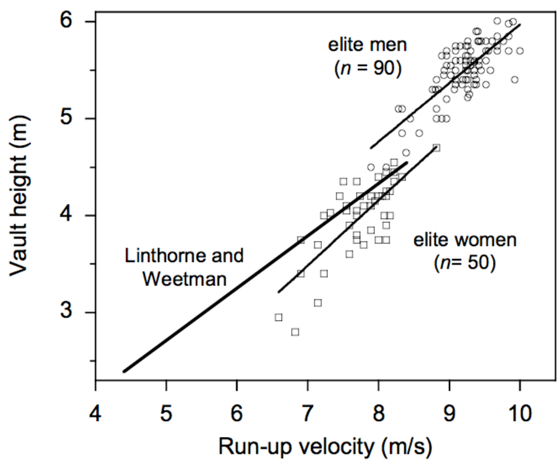

Figure 4: This plot shows the strong correlation between vault height and run-up velocity within groups of elite pole vaulters. Figure from [10]. We may at first sight think that the vault height should be proportional to the square of the run-up velocity, but we see a linear law emerging, this is the trace of energy losses at the planting shock through inelastic response of the body.

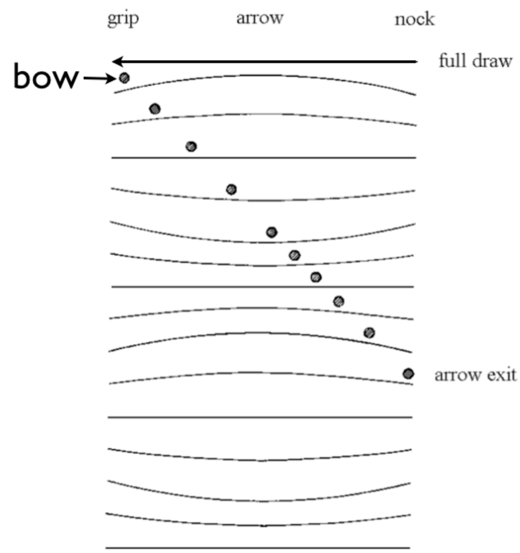

Figure 5: The Archer's paradox. Dynamic bending of the arrow right after being released from the archer's finger. The arrow oscillates and this oscillation must be tuned such as to avoid collision with the bow. Figure from [12].

stiff, but it evolved into bamboo, steel, then glass fibers and nowadays carbon fibers. Since the introduction of well tuned soft poles, the athlete may ideally hope to convert all of his kinetic energy into height, and possibly even more since he can use his power during the flight. This discipline is very technical and ask for both strength and speed but as well a large gymnic ability, so that champions of this sport are typically older than other athletic disciplines; it takes time to master the interaction with the bending and recoil of the pole during flight. To clear the highest crossbar possible, the length and stiffness of the pole must be carefully fit to the strength and gymnic ability of the vaulter.

We saw how the long jumper was facing a dilemma for choosing his take-off angle. The case of the pole vaulter is yet more subtle. If the specific requirements of the trajectory dynamics did allow the pole to be extremely soft, there would be no shock during the planting of the pole and take-off. Thus, in light of what we mentioned for long-jump, the take off angle should be zero degree: the vaulter would completely rely on the elasticity of the pole to redirect upward his horizontal velocity. But instead, vaulters choose poles much stiffer than this, because the long trajectory with a soft pole is too sensitive to precision of the jump: it is much easier to control the trajectory from the take-off and having a quick fly-away than adjusting actively the trajectory while flying on a long trajectory. For this, see the contribution of J. Hoepffner "Models for an alternative pole vault" [11]. Thus athletes chose a relatively stiffer pole and have to suffer a take-off shock. The long-jumper has a shock in its leg as a pivot, and the pole vaulter has in addition a shock in the upper body when planting his pole. The take-off angle here is the choice wether to loose energy through a shock in the leg or in the upper body, and this choice depends of course also on the stiffness of the pole: an intricate situation. Vaulters loose an energy at take off which is typically equivalent of about 2m/s of running speed. The final height which can be cleared will critically depend on how much of this energy can be saved. The contribution of N. Linthorne [10] discusses these issues through experiments with skilled vaulters using as control parameter the run-up velocity. Some of his data is shown in figure 4, showing experimentally that the performance depends approximately linearly on running speed at take-off. This situation seems at first paradoxical since the kinetic energy depends like the square of the running speed.

5 Elasticity and shape deformation

In the previous sections, I have mainly discussed the importance of elasticity in the inside of the body. For pole vault, there is an additional elastic device outside of the body, and we saw that because of the planting shock, these two elastic behavior interact and the choice of the take-off angle will depend of this interaction. In the present section, I present a few typical aspects of elasticity outside of the body.

One first interesting example is that of the Archer's paradox, the observation that when shooting an arrow with a bow, the initial direction of the arrow should not be that of the aim, but a little to the left for a right handed archer.

Figure 6: Impact of a squash ball right at a corner. Image from [13]. The ball shows a strong elastic deformation. The bounce at the corners leads to a large dissipation of energy, due to viscous loss in the deformation of the shell dry friction at the contact surface between the deforming shell and the surface.

Indeed, when the archer releases the string with his fingers from the right side, there is first a quick motion to the left which induces a strong bending of the arrow. This arrow thus gain forward momentum while oscillating along its length with a period of oscillation related to its bending rigidity and mass. The situation is shown in figure 5. The bending rigidity of the arrow should be fit to its mass and translation velocity such that it does not hit the bow. See [12].

When bouncing, a ball has ideally the ability to change direction at constant total energy, the kinetic energy of initial translation is stored into elastic energy during contact with the surface: compression of air inside the ball and elastic deformation of the shell. This bounce is passive, there is here no source of new energy. Energy losses are made by viscoelastic behavior of the shell, heating and slight heat diffusion of the compressed air inside, but also friction of the deformed shell against the wall: the shape is spherical at rest and must deform to a plane about the contact point. The progressive squashing of the ball and transformation from the sphere to plane leads to sliding of the elastic surface against the contact surface and thus to dry friction. We can as well see the effect of this dry friction with elastic deformation for wheel-road interaction for the inflated wheels of our cars, see [14].

A related situation is found in the contribution of P. Brunet, [13], "The (sometimes) strange bounce of a squash ball". It addresses a trick called "the nick" which consist of having the ball bounce right at the corner made by two perpendicular walls. In that situation, the loss of energy is large so the ball almost does not bounce. The interesting question here consists in asking wether the loss of energy comes from enhanced viscoelastic behavior of the shell because it has to deform more at a corner, or if it has to do with surface friction. Indeed, when bouncing at a corner, the ball has first two contact points with the two surfaces, and these points must slide while the ball elastically deform and is being squashed inside the corner by inertia. The sliding motion is made again in reverse direction while the ball regains its spherical shape and moves away from the corner. A photograph of the deformed squash ball bouncing at a corner is shown in figure 6.

An other instance of a ball bouncing is discussed in the contribution of L. Smith, "Baseball-bat performance" [15]. The question is to understand how the ball looses energy while bouncing against the bat. It is shown that experiments with static bat overestimate the damping due to friction inside the ball and at the ball/bat contact surface. The main difference between this static experiment and the dynamic case of a moving bat is the elasticity and vibrating motion of the bat. At impact, the elastic bat deforms, accumulates some of the kinetic energy from the incoming ball, and vibrates along its own elastic behavior: rigidity and mass. The vibration may be tuned such as to allow restitution of the energy. In the static situation, all of the elastic deformation happens inside the ball, with possibly a large internal friction. With a flexible bat, part of this deformation happens in the bat instead, with a lower damping since the material can be freely chosen. The choice of the right bat will thus depend on the ability of the elastic deformation to give back the energy to the ball at the right time.

We saw another interesting sport involving elasticity in the contribution of A.L Biance and E. Reyssat, "Physics on a slackline", [17]. This popular side-activity of rock climbers consists in walking and shaking on a slack elastic rope. The contribution considers how the slackness affects the growth rate of the instability of an inverted pendulum standing on the slackline. During the conference, the authors showed videos of skilled slackers using a strategy of periodic motion of large amplitude involving extension of the line. We know examples of systems where an elastic instability, typically buckling, can be stabilized through a periodic excitation. In see for instance [16], a flexible wire is clamped upward. This situation is unstable to buckling of the wire under its own weight if its length exceeds a critical value. If the attachment position of the wire is oscillated vertically, there are windows of frequencies for which the system is rendered stable. This phenomenon is known in the literature as the "indian wire trick". We may speculate that an analog mechanism may be used to stabilize the slacker on its slackline.

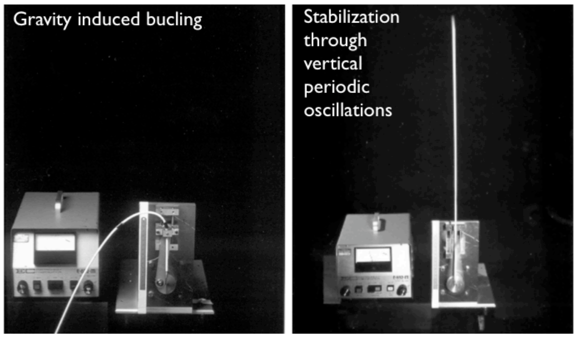

Figure 7: The ”indian wire trick”. A wire is held clamped upwards. a) without excitation the wire buckles under its own weight and falls down on the side, whereas in b), the attachment point is oscillated vertically and the upward position is stabilized. Figure from [16]. This example shows how a periodic excitation can stabilize an elastic system by making use of its elastic properties of oscillations.

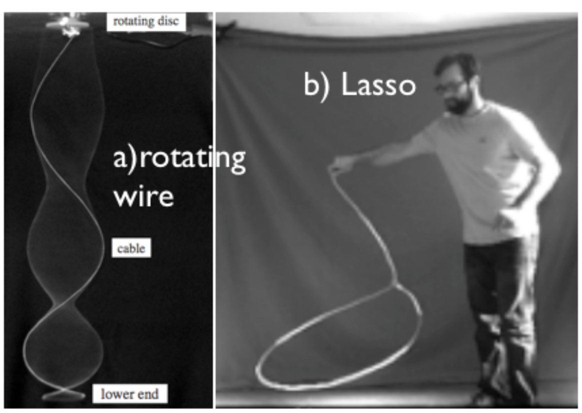

Figure 8: a) A wire in a rotating frame may adopt different steady configurations under the effect of the centrifugal force: a force that depends on the position of the wire with respect to the axis of rotation. b) The steady shape of the lasso. Figure adapted from [18] and [19].

The conference gave us yet another occasion to observe the effects of elasticity in sports: the shape and stability of the lasso, in the contribution of P. T. Brun ”how does a physicist catch a cow”, [19]. We can first think of the work of [18] in which a heavy wire is held hanging from a rotating support. In the reference frame rotating with the support there are several stable steady solutions with increasing number of peaks, as shown on figure 8. This wire is subject to a very specific force along its length: the centrifugal force which is a function of the distance between the wire and the axis of rotation. The steady shape is a result of a force due to its motion. The case of the lasso is analogous, except that it has a closed loop.

6 Conclusion

Common knowledge about sports tells that we must train our muscles. These muscles often must be quick, powerful and precise. This view comes from a conception of body motion in the spirit of imperative control: the body has a given bone structure and the muscles are here to activate this structure to the desired discipline-specific moves, accounting for inertia of the limbs. We know now that in fact muscles and tendons have essential properties of elasticity: the body is activated by the contraction of the muscles, in interaction with the inner elasticity of the structures. The properties of this elasticity can be specifically trained in order for the body reaction to have elastic response tuned to the strength and time scales of the specific required moves. Performance in many instances is not only related to the production of energy itself but also to the economy of this energy.

The way how elastic response has to be tuned to the move is also illustrated in the Archer’s paradox: the arrow bends while being accelerated and the period of oscillation must be tuned to the time it takes to pass along the bow, lest collision occurs. The baseball can advantageously bounce on the bat if the bat deforms and restore the energy with the right timing: tuning of the bat elasticity and the ball elasticity.

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