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Control of instabilities in a cavity-driven separated boundary-layer flow

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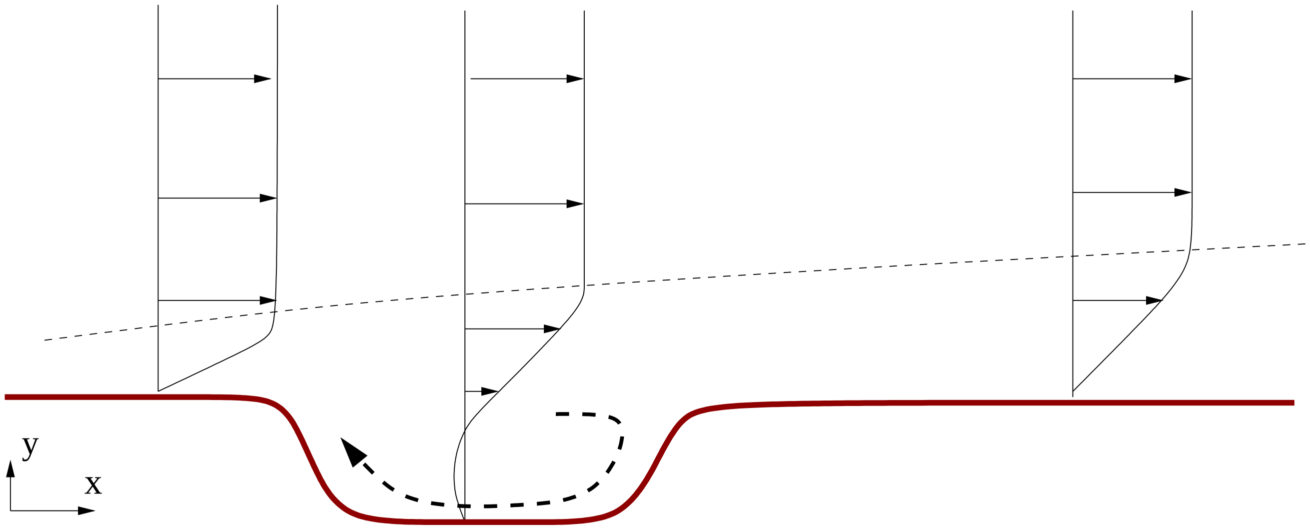
Boundary layer with cavity

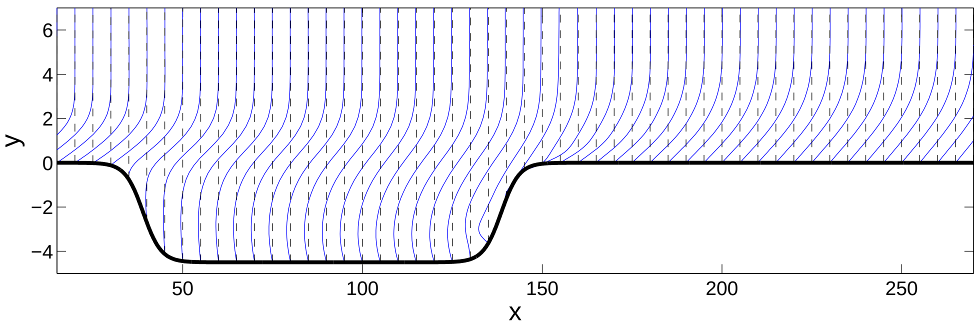


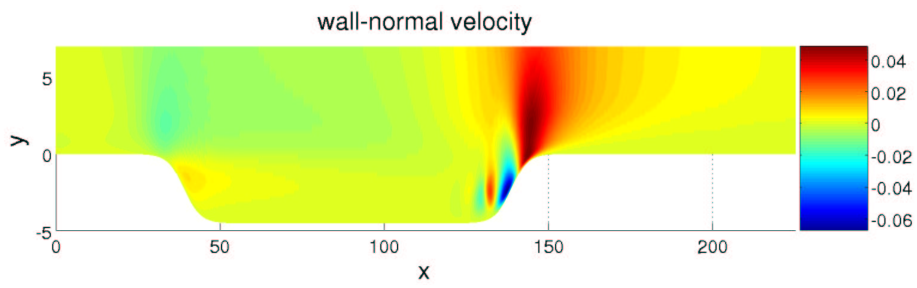
2D flow over a smooth cavity

Long aspect ratio: 20

Inflow: Blasius profile

Reynolds number : 325





Investigation tools

Flow description: **DNS** to compute the base flow:
Chebyshev in wall normal, finite difference in streamwise.
Stability analysis by computation of 2D eigenmodes:
Chebyshev/Chebyshev and Arnoldi

From eigenmodes: **Optimal growth** by optimization over initial conditions :
Singular value decomposition
Control optimization by solution of two Riccati equations

The eigensolver

2D Navier-Stokes + continuity

$$\begin{cases} -i\omega\hat{u} = -(U\cdot\nabla)\hat{u} - (\hat{\mathsf{u}}\cdot\nabla)U - \dfrac{\partial\hat{p}}{\partial x} + 1/Re\nabla^2\hat{u} \\ -i\omega\hat{v} = -(U\cdot\nabla)\hat{v} - (\hat{\mathsf{u}}\cdot\nabla)V - \dfrac{\partial\hat{p}}{\partial y} + 1/Re\nabla^2\hat{v} \\ 0 = \nabla\cdot\mathsf{u} \end{cases}$$

Generalized eigenproblem:

$$B\omega\mathsf{u} = A\mathsf{u}$$

To be rewritten

$$A^{-1}B\mathsf{u} = \frac{1}{\omega}\mathsf{u}$$

Solved by Arnoldi iterations.

Matrix formulation:

$$\begin{pmatrix} -i\omega\hat{u} \\ -i\omega\hat{v} \\ 0 \end{pmatrix} = \begin{pmatrix} \dots & \dots & -\frac{\partial}{\partial x} \\ \dots & \dots & -\frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & C \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{v} \\ p \end{pmatrix}$$

Pressure constraints C

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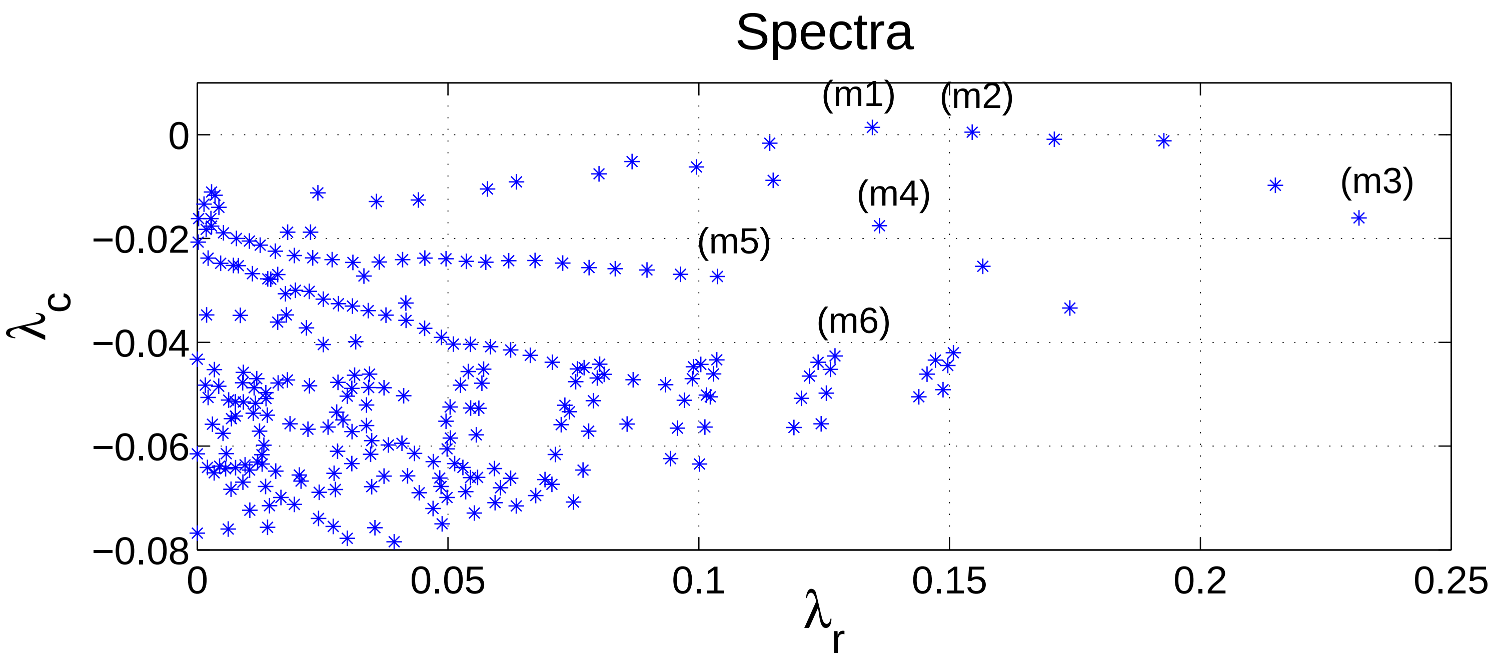
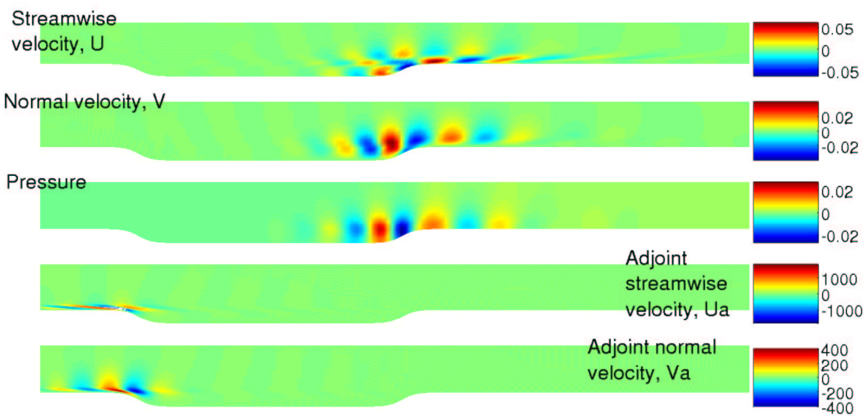
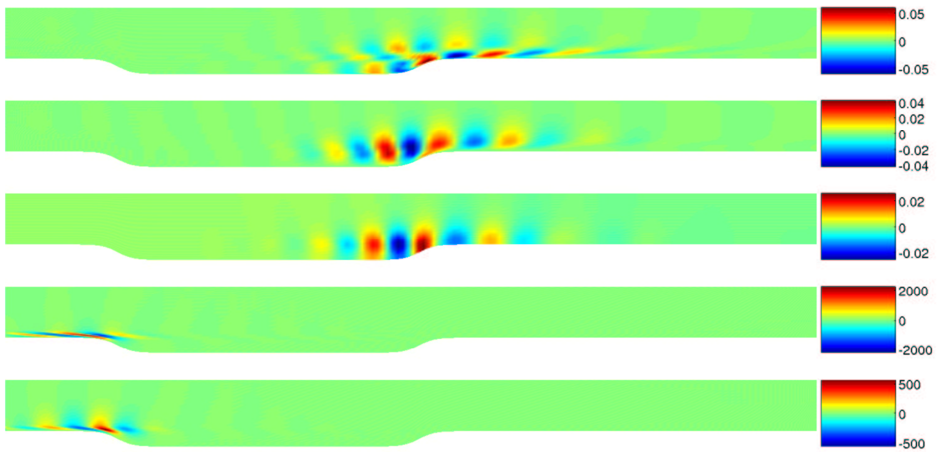
Eigenmodes







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Optimal growth from initial conditions

System $x(t) = Ax, \quad \dot{x}(0) = x_0$, with solution

$$x(t) = e^{At}x_0$$

Find the initial condition x_0 maximizing

$$G(t) = max_{x_0} \frac{< x(t), x(t) >}{< x_0, x_0 >},$$

adjoint: $< Ax_1, x_2 >=< x_1, A^+ x_2 > \forall x_1, x_2$

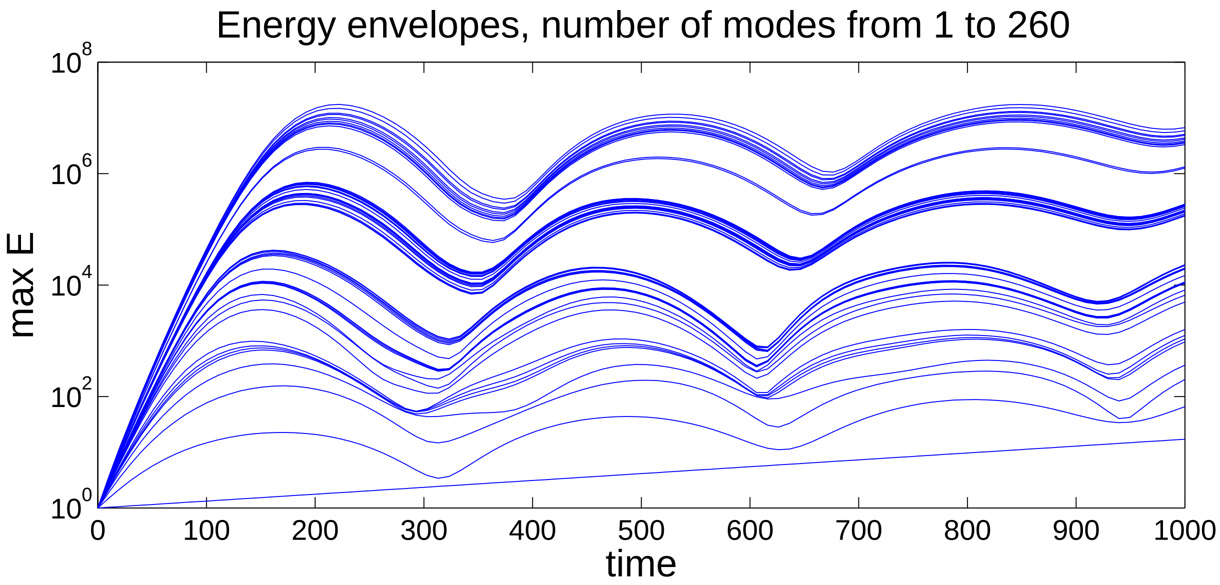
leads to

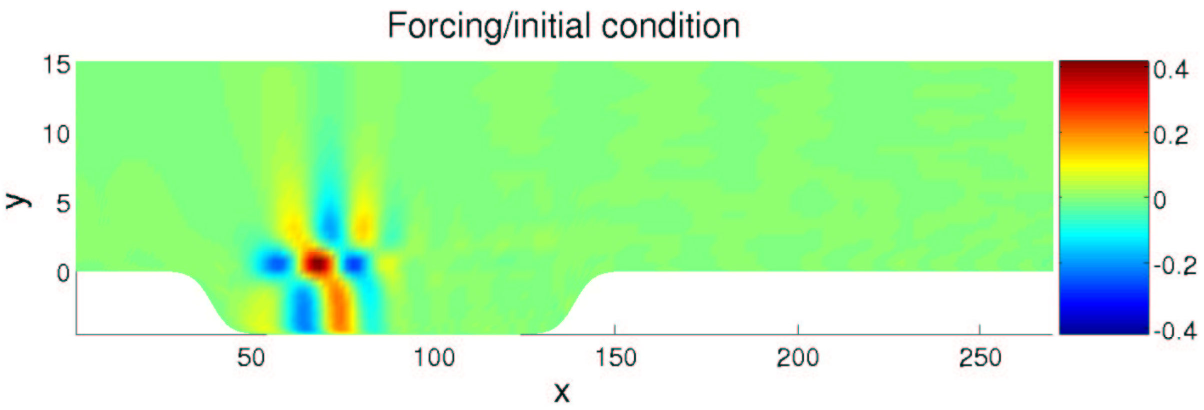
$$G(t) = max \frac{< e^{At}x_0, e^{At}x_0 >}{< x_0, x_0 >} = max \frac{< e^{A^+t}e^{At}x_0, x_0 >}{< x_0, x_0 >}$$

$\longrightarrow$ Maximum growth at time t: eigenvalue of $e^{A^+t}e^{At}$.

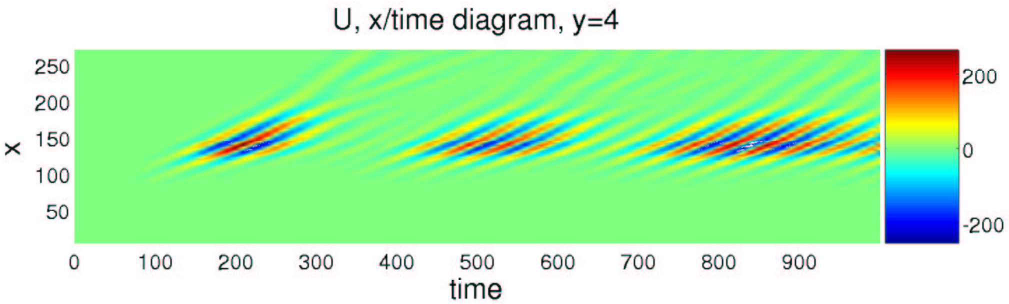
Optimal growth in the cavity

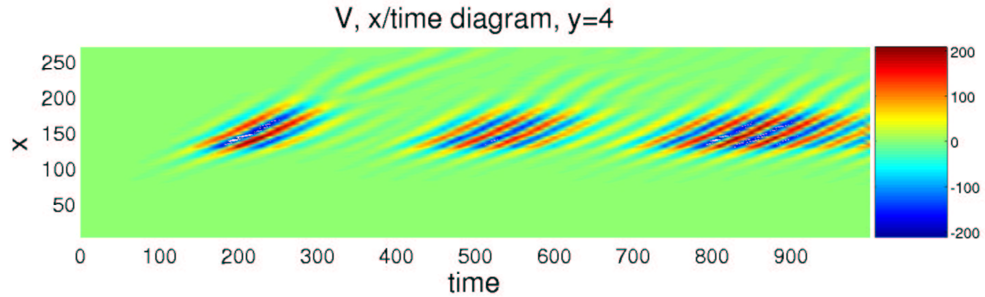
- Global instability
- Potentiality of strong energy growth
- Low frequency cycle





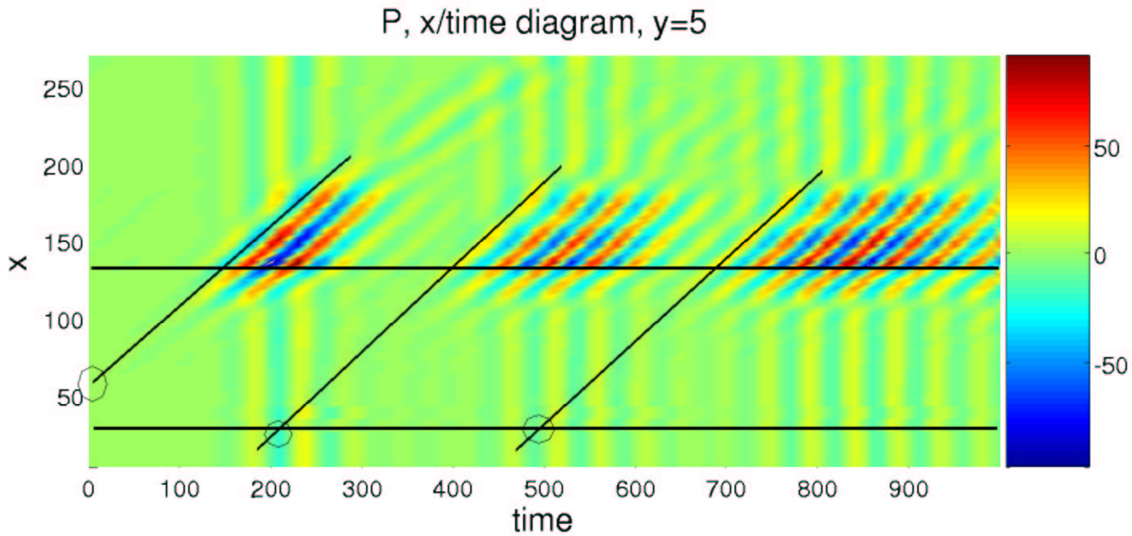
Flow cycle





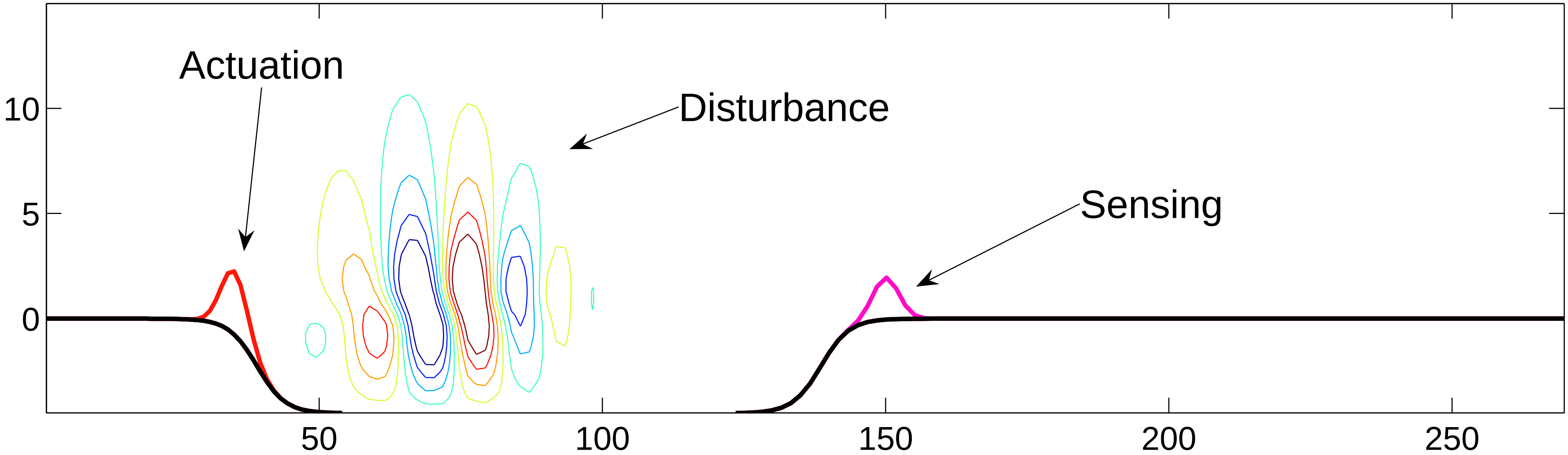
Generation of **global pressure change** when the wave-packet impacts on the downstream lip

Regeneration of disturbances when the pressure hits the upstream lip



Control

Seek to minimize the energy growth



- One actuator upstream
- One sensor downstream
- Oscillating disturbance in the shear layer

Feedback control

Dynamic model: $\begin{cases} \dot{x} = Ax + Bu \\ r = Cx \end{cases}$ Optimize for the feedback $u = \mathcal{G}(r)$

- The model in 2D is too big for optimization $\rightarrow$ reduced model .
- For reduction: project the dynamics on the least stable eigenmodes.
- Finally, couple the reduced controller and the flow system

Estimation: estimate flow state from sensors.

Control: Actuate from feedback of estimated flow.

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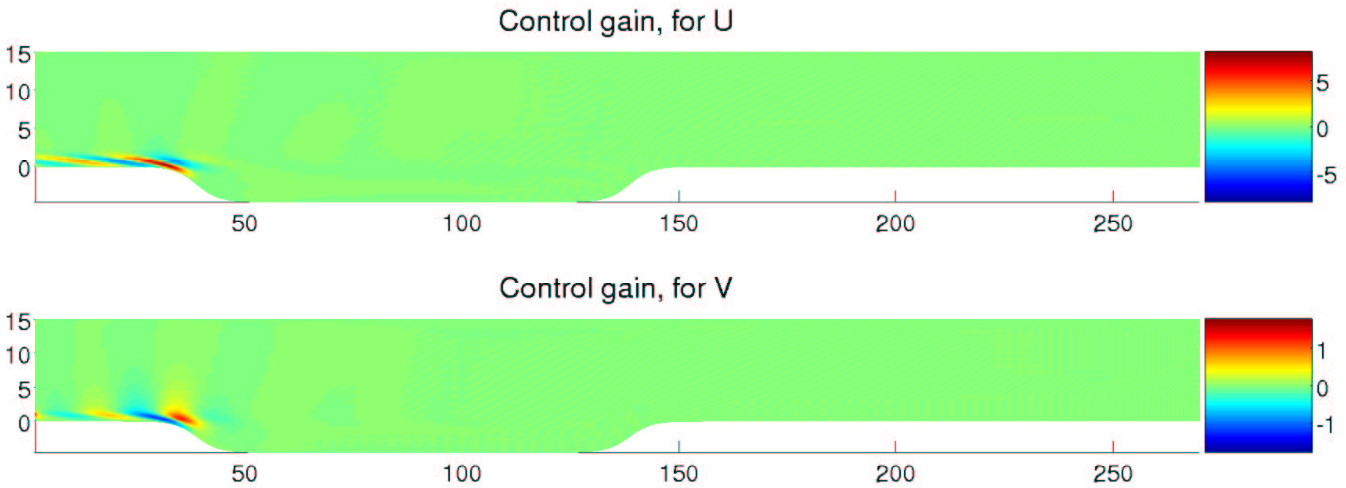
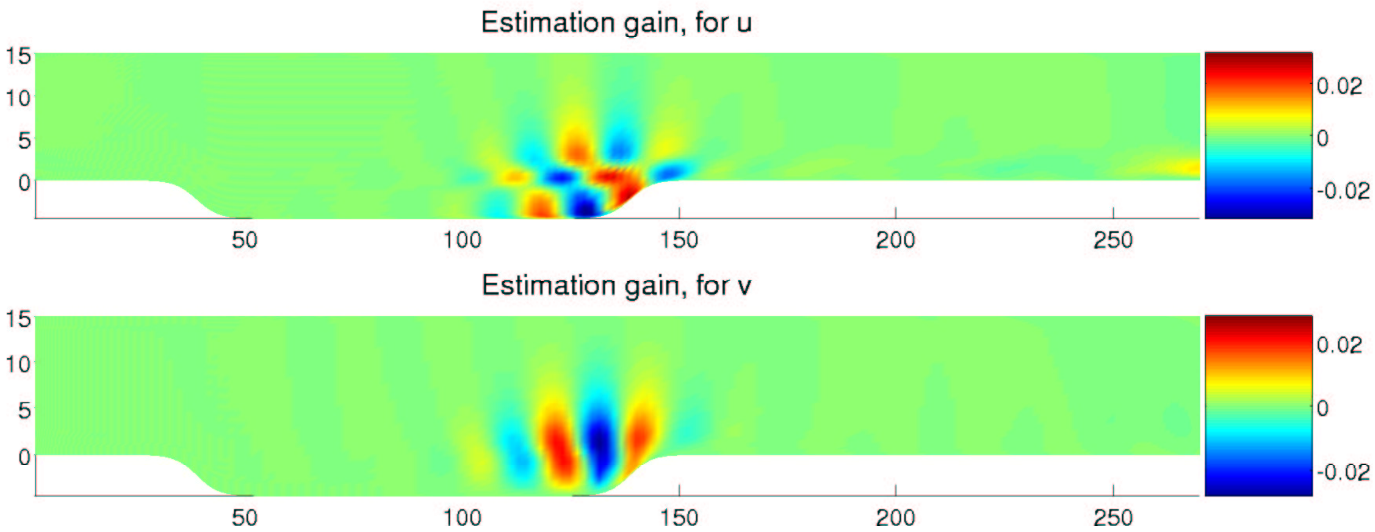
Control and estimation gains

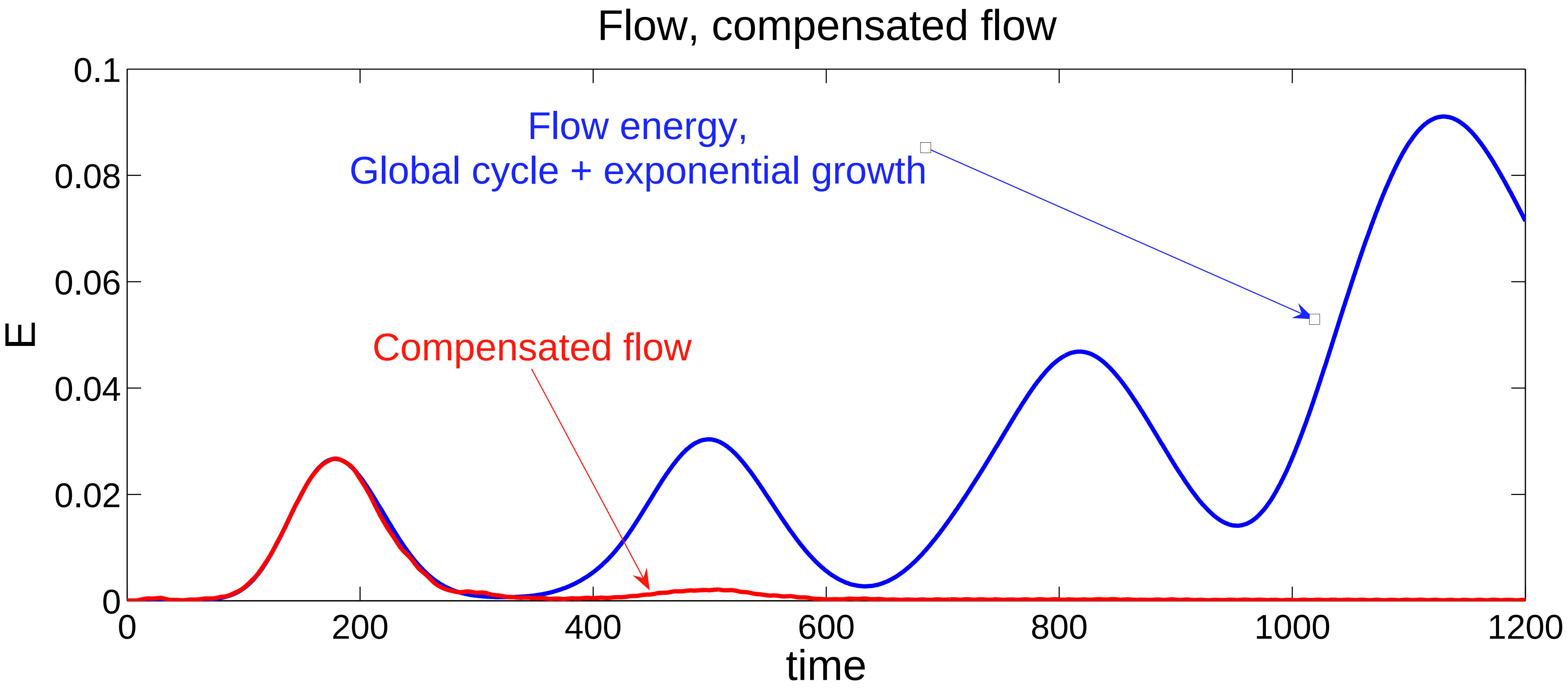
Function used to **extract the actuation signal** from the flow



Function used to **force the estimator** flow



Compensation performance



Good control performance from the second cycle

Reduced order model:20 states

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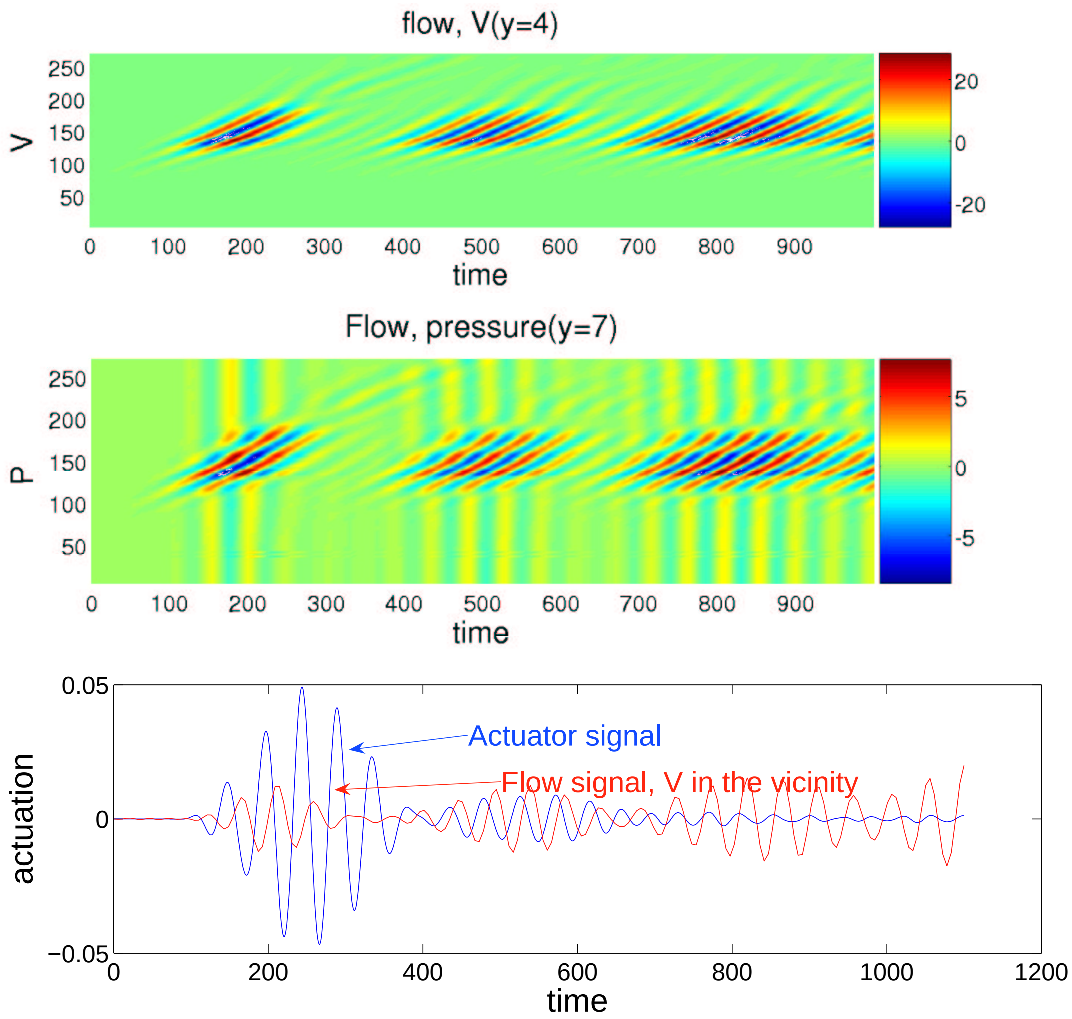
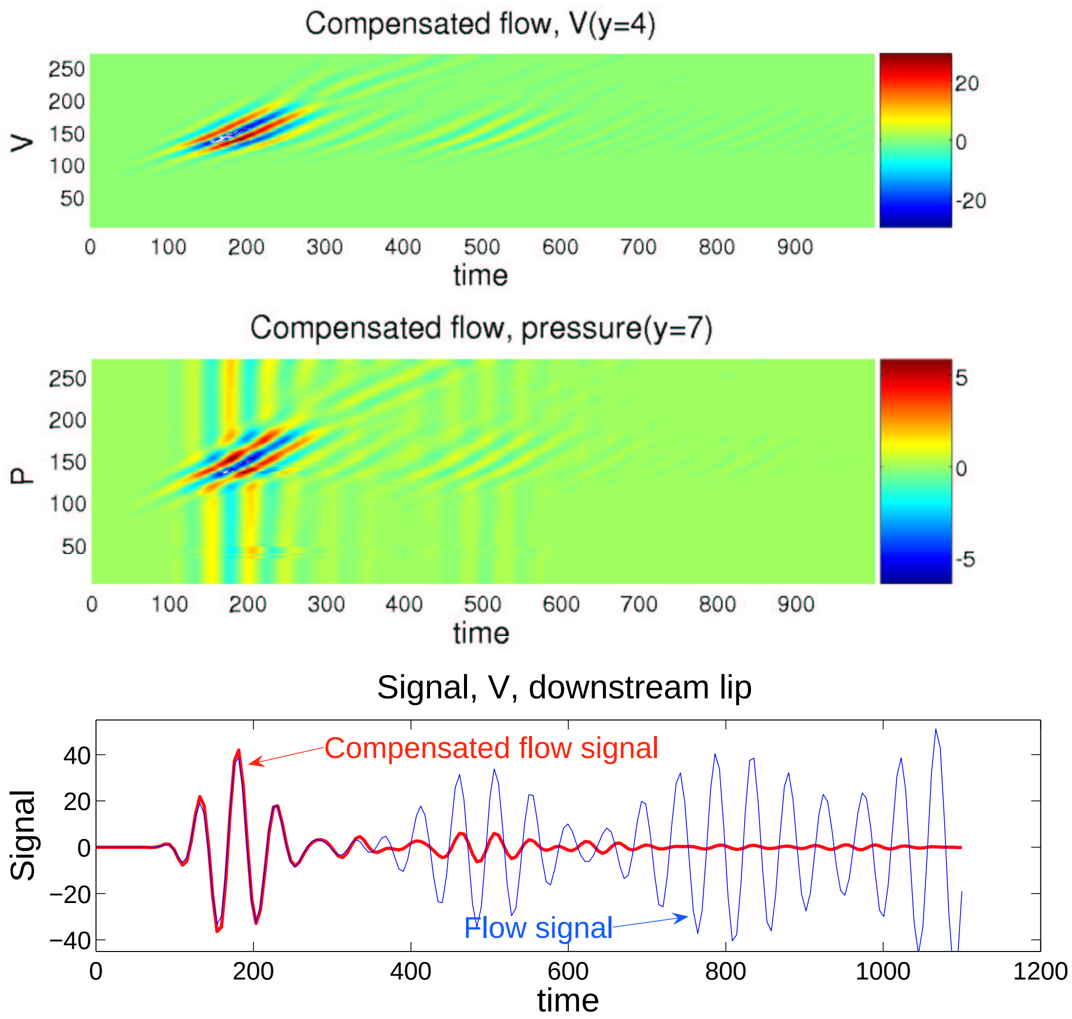
Flow



compensated flow



Conclusion

Flow dynamics:

- Incompressible cavity can have global cycle due to pressure.

Global modes:

- Global eigenmodes can be used for analysis and model reduction.
- Convective instability well described by non-normality of global modes

Control:

- Model reduction allows optimal feedback design for large systems.
- Non-parallel effects/global instabilities can be treated.

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Extra slides

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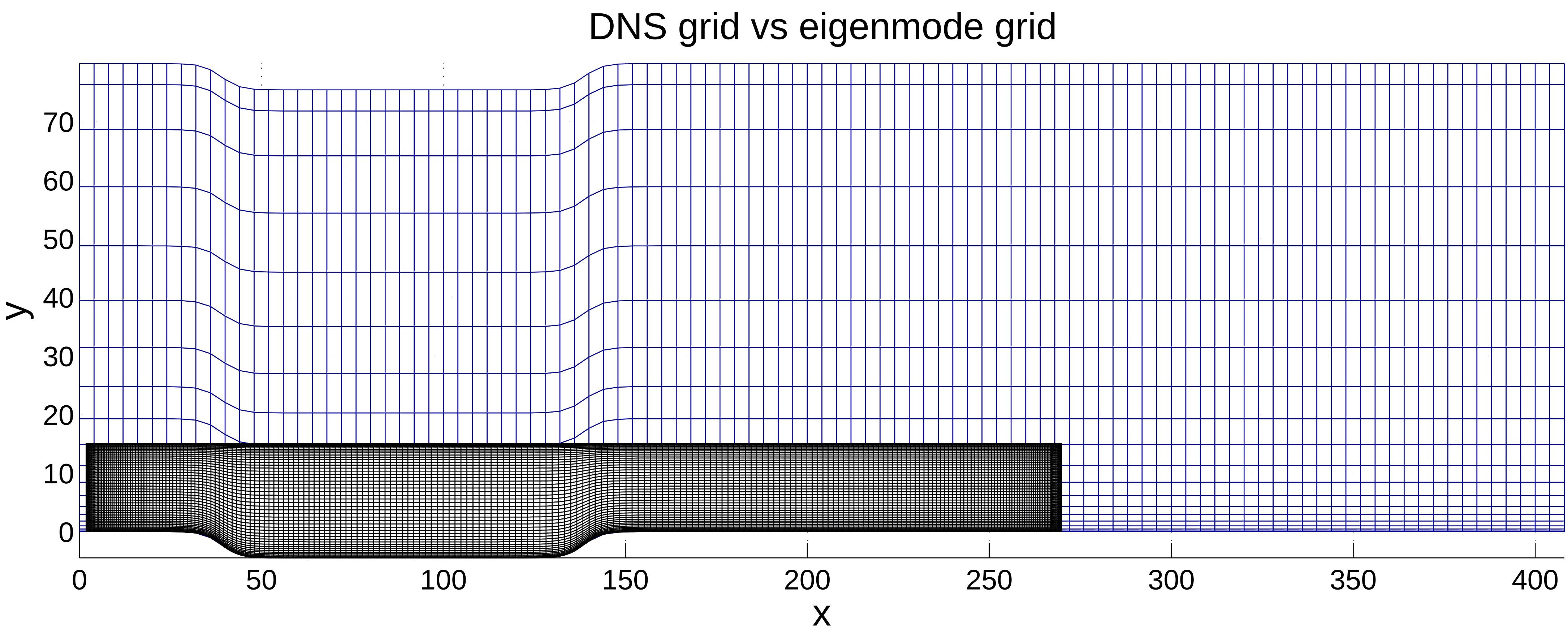
Grids & resolution

The resolution are:

DNS: nx=2048 finite difference, ny=97 Chebyshev, Lx=409, Ly=80

EIG: nx=250 Chebyshev, ny= 50 Chebyshev, Lx=270. Ly=15.



Control terminology

- **Estimation:** From sensor information, recover the instantaneous flow field.
- **Full information control:** From full knowledge of the flow state, apply control.
- **Compensation:** Close the loop by using the estimated flow state for control.
- **Model reduction:** Project the dynamics on a set of selected basis vectors.

- **Control penalty:** Penalisation of the actuation amplitude.
- **sensor noise:** Uncertainty in the measured signal.

- **Disturbances:** External forcing exciting the flow.
- **Objective function:** Function of the flow state to be minimized.

Model reduction

Galerkin projection on least stable eigenmodes:

Physical space:

$$\begin{cases} \dot{x} = Ax + Bu \\ r = Cx \end{cases}$$

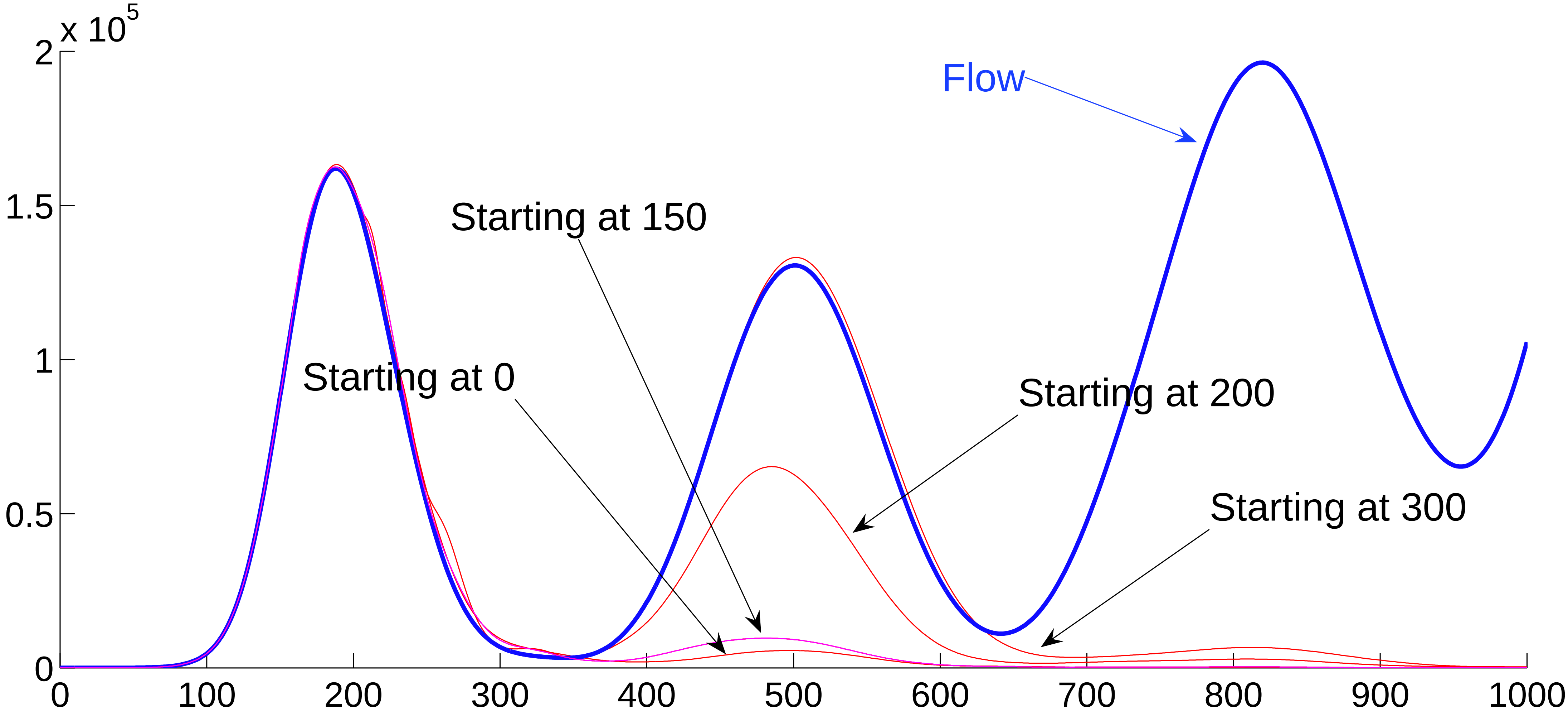
Eigenmode space:

$$\begin{cases} \underbrace{P\dot{x}}_{\dot{k}} = \underbrace{PAP^{-1}}_{A^M} \underbrace{Px}_{k} + \underbrace{PB}_{B^M} u \\ r = \underbrace{CP^{-1}}_{C^M} \underbrace{Px}_{k} \end{cases}$$

Projection on eigenmodes → **biorthogonal** set of vectors:

- Eigenmodes: q_i,
- Adjoint operator: $A^+ / < Ax_1, x_2 > = < x_1, A^+ x_2 >, \forall x_1, x_2$
- Adjoint eigenmodes: q_i^+,
- Biorthogonality: $\delta_{ij} = < q_i, q_j^+ >$, Projection: $k_i = < x, q_i^+ >$

Starting the compensator at later times



Compensator cannot affect the disturbance propagation

but can affect the disturbance generation

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Dynamic distortion

blue :flow

Red :compensated flow